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APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATION OF TIME DELAY  
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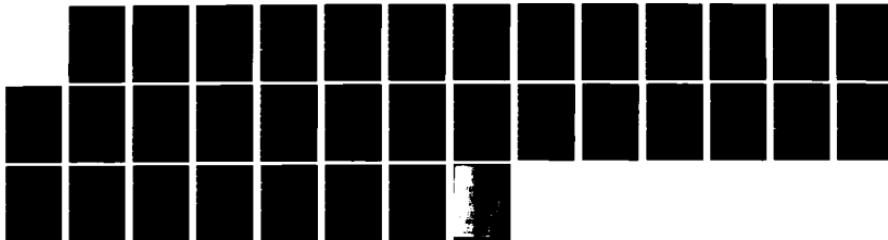
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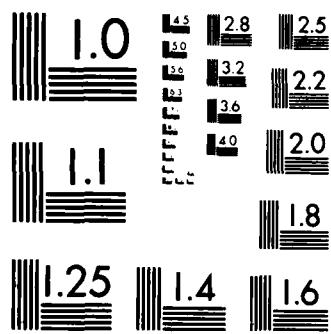
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**AD A 12837**  
**APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATION OF TIME DELAY IN  
DETERMINISTIC AND STOCHASTIC SIGNAL PLUS NOISE MODELS**

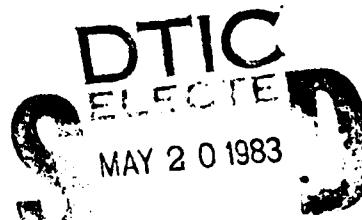
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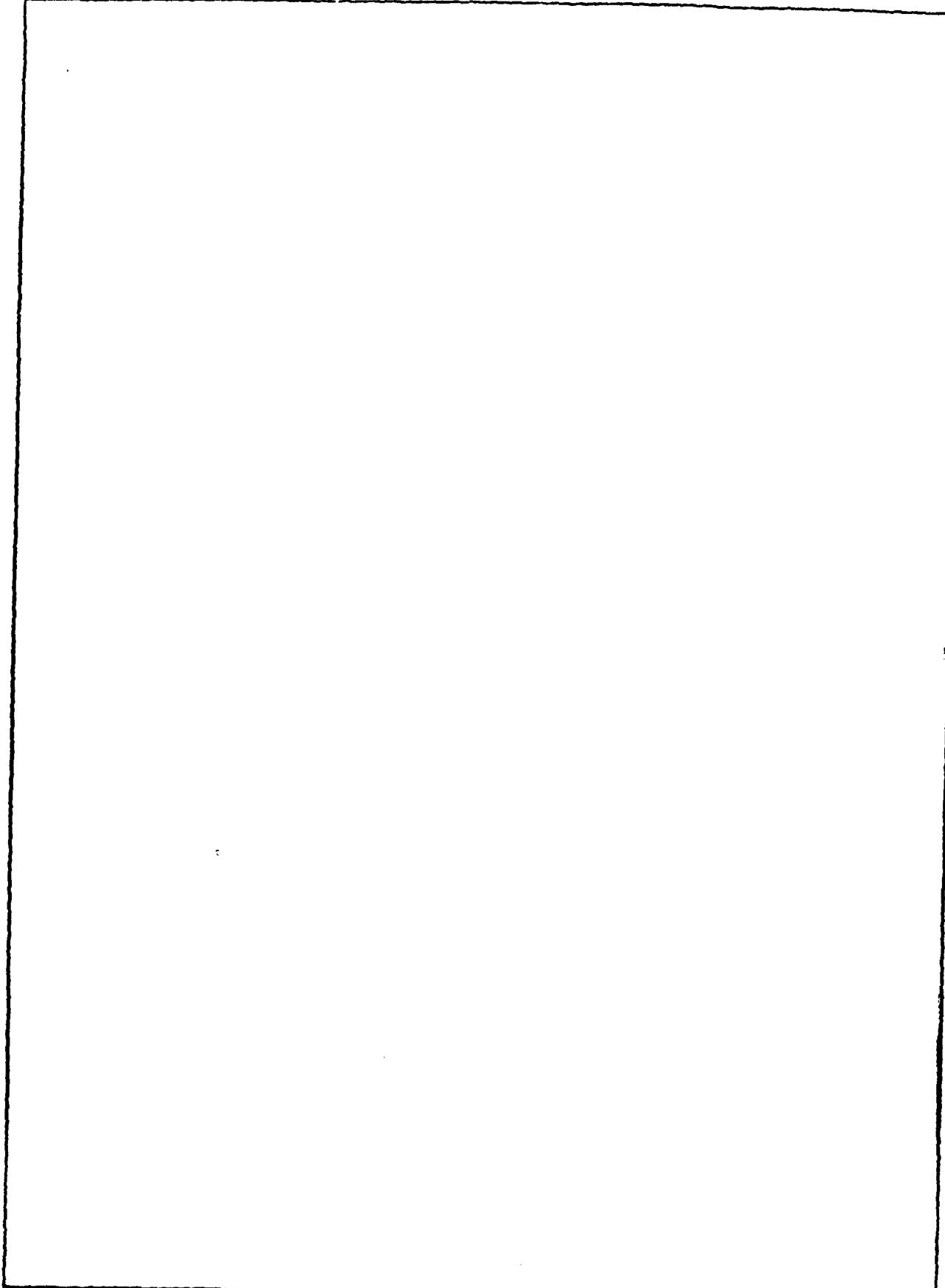
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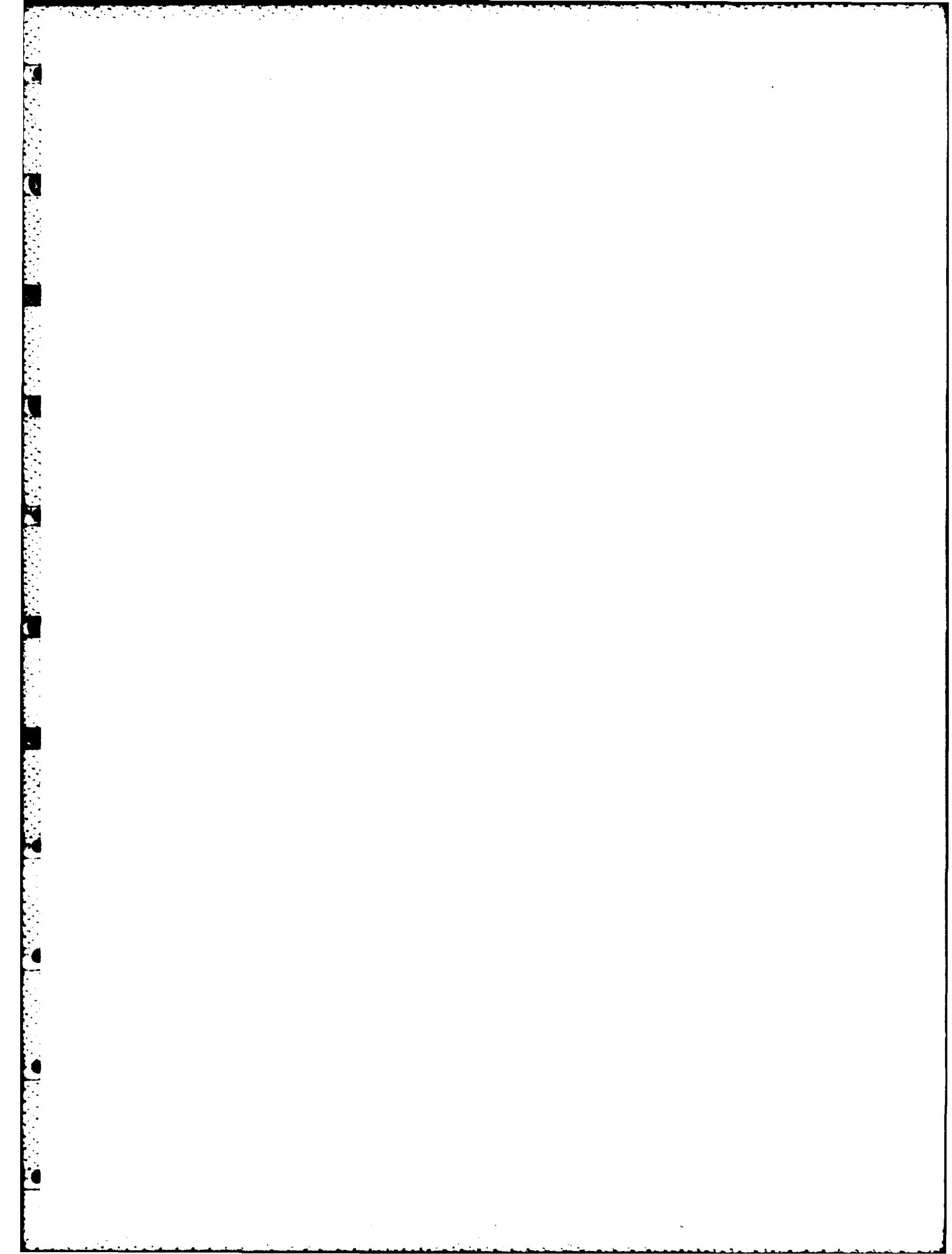
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Abstract

The problem of estimating signal delay in signal-plus-noise models is considered. A technique is developed for approximating likelihood in such a way that a dynamic programming algorithm may be used to efficiently and recursively estimate delay. The approximation provides an alternative to the implementation of a growing bank of matched filters for exhaustive maximization of likelihood. Simulation results indicate acceptable performance at output SNRs in excess of 15 dB for deterministic signals and in excess of 20 dB for stochastic signals.



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## INTRODUCTION

The estimation of arrival time or event time in sonar and radar applications is such a commonly encountered problem that it has acquired its own acronym: TDE (for time delay estimation)[1]. There are however other applications where an estimate of event time is relevant. These include highway traffic control[2] and fault detection[3].

Solutions to the TDE problem vary in sophistication from threshold detection as used by Iyer[4], to tests of whiteness, means and covariances employed by Mehra and Peschon[5], to application of the generalized likelihood ratio by Willsky and Jones[6]. Iyer derives a threshold detection algorithm which detects the rising edge of satellite broadened laser pulses. He models the arrival time of the pulse as a Poisson process. Based on this he derives the probability that the threshold value is exceeded at some time  $j$ . He then maximizes this probability with respect to the arrival time to obtain the detection time.

Mehra and Peschon outline an innovations approach to fault detection. They propose hypothesis tests on whiteness, means, and covariances of the innovations sequence of a Kalman predictor. This approach is applicable in those applications which allow batch or fixed lag processing of the observations.

Willsky and Jones use a generalized likelihood ratio test to detect and estimate jumps in the state variables in autoregressive moving average (ARMA) systems driven by white noise. They model a jump in the state of the system as a single pulse whose amplitude and occurrence time are unknown parameters. They note that implementation of their algorithm requires a growing bank of matched filters. This is due to the fact that likelihood must be computed for a jump occurrence at each time from time 0 to the present. They

suggest an approximation in which likelihood is computed only for the current and previous M-1 occurrence times.

The problem of growing banks of filters to implement TDE algorithms is common. Another method that is often used to avoid this problem in multi-sensor applications is that employed by Kennefic[7]. He uses the N scan approximation first developed by Singer et al.[8] to efficiently implement an algorithm which generates the a-posteriori density function of a Markov delay. In this case all the filters with an identical history for the past N steps are replaced by a single filter.

In this paper we present another method of approximation for avoiding the use of a growing bank of filters. The method used is to compute the likelihood of delay on a subset of the interval from the initial time to the present. The subset is defined recursively, in a forward dynamic programming algorithm.

The TDE algorithm presented here is dynamic and the formalism in which we present it is much the same as that normally associated with dynamic programming. The algorithm does not yield the optimum solution to the TDE problem. The algorithm is however recursive in nature, and storage requirements are constant in time. There is no need for a growing bank of filters.

TIME DELAY ESTIMATION IN DETERMINISTIC AND  
STOCHASTIC SIGNAL MODELS

In single sensor time delay estimation problems, we are interested in estimating the arrival time of a signal. The signal may be deterministic or stochastic. Here we introduce an observation model and two signal models. We give several equivalent characterizations of the TDE problem and derive the joint distribution of the observations for both the deterministic and stochastic signal models.

Observation Model

Consider the discrete-time sequence  $\{y_t : t = 0, 1, \dots, T-1\}$ . A noisy, delayed version of the sequence is observed:

$$z_t = u_t^{(d)} y_{t-d} + n_t, \quad t = 0, 1, \dots, T-1$$

Here  $d$  is the delay and  $\{n_t : t = 0, 1, \dots, T-1\}$  is a sequence of independent identically distributed (iid) normal random variables with mean zero and variance  $\sigma_n^2$ . The sequence  $\{u_t^{(d)}\}_{0}^{T-1}$  is an indicator sequence:

$$u_t^{(d)} = \begin{cases} 0 & 0 \leq t < d \\ 1 & d \leq t \leq T-1 \end{cases}$$

The indicator sequence characterizes the delay.

Time Delay Estimation Problem

Let  $I_t$  denote the discrete set  $I_t = \{0, 1, \dots, t+1\}$ , and let  $Y_t$  denote the observations to time  $t$ :  $Y_t = \{y_k\}_0^t$ . Call  $\hat{d}_t$  an estimator of  $d$  that maps observations  $Y_t$  into the set  $I_{t+1}$ :

$$\hat{d}_t: Y_t \rightarrow I_{t+1}$$

When  $\hat{d}_t = t+1$ , we say the delay is estimated to be greater than  $t$ , indicating that the observations  $Y_t$  are noise only. Typically we construct a sequence

of estimators  $\hat{d}_0, \hat{d}_1, \dots, \hat{d}_t$  that terminates at time  $t = T-1$  in an estimate  $\hat{d}_{T-1}$  based on the complete observation set  $Y_{T-1}$ . A typical sequence would look like this:

t:	0	1	2	3	4	5	6	7
$\hat{d}_t$ :	1	2	3	4	5	5	5	5

In this case we would say  $\hat{d}_{T-1} = 5$ . Note that for  $t \leq 4$ , the data is judged to be purely noise.

With this formalism we may give three equivalent statements of the time delay estimation problem: given the observations  $Y_t$ ,

- (1) find the maximum likelihood estimate of  $d$ , denoted  $\hat{d}_t$ , under the constraint that  $\hat{d}_t$  lie in  $I_{t+1}$
- (2) find the most likely indicator sequence  $\{u_k(\hat{d}_t)\}_{0}^t$
- (3) find the minimum probability of error test of  $H_0$  vs.  $H_1$  vs. ... vs.  $H_t$  vs.  $H_{t+1}$ , with  $H_k$  denoting the hypothesis that  $d = k$ ,  $k \in I_{t+1}$ ; the understanding is that  $d = t+1$  corresponds to the noise-only hypothesis.

Here's the sense of the equivalence. Fix  $t$ . Let  $\pi(H_k) = 1/(t+1)$  denote the prior distribution over the  $H_k$ . As the sequence  $\{u_k(\hat{d}_t); k = 1, 2, \dots, t\}$  maximizes likelihood it follows that  $\hat{d}_t$  is the choice that minimizes the probability of error in selecting an hypothesis. And, as there is a 1:1 correspondence between  $\hat{d}_t$  and the sequence  $\{u_k(\hat{d}_t)\}$  it follows that  $\hat{d}_t$  is also the maximum likelihood estimate of  $d$  under the constraint that  $\hat{d}_t$  lie in the set  $I_{t+1}$ .

#### Deterministic Signal

If the signal sequence  $\{y_t\}_{0}^{T-1}$  is known, then the distribution of the noisy sequence  $\{z_k\}_{0}^t$  is

$$f\{z_k\}_0^t = \prod_{k=0}^{\max(d-1,0)} (2\pi\sigma_n^2)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma_n^2} z_k^2\right\} \prod_{k=\min(d,t)}^t (2\pi\sigma_n^2)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma_n^2} (z_k - u_k(d)y_{k-d})^2\right\}$$

It is assumed that only the delay parameter  $d$  is unknown.

### Stochastic Signal

Many physical processes may be effectively modeled as stationary autoregressive moving average (ARMA) systems driven by white noise. The Markovian state space model for such a process is

$$x_{t+1} = Ax_t + hw_t$$

$$y_t = c'x_t + w_t$$

where  $\{w_t\}$  is a sequence of iid normal random variables with mean zero and variance  $\sigma_w^2$ , and the matrix triple  $(c', A, h)$  is characterized as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & & \ddots & & & \\ \vdots & & & & 1 & \\ -a_p & \dots & \dots & \dots & \dots & -a_1 \end{bmatrix} \quad h = \begin{bmatrix} h_1 \\ \vdots \\ h_p \end{bmatrix}$$

$$c' = [1, 0, \dots, 0]$$

The impulse response vector  $h$  is determined by

$$\begin{bmatrix} 1 \\ a_1 \\ \vdots \\ \vdots & 1 \\ a_p & \dots & a_1 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} 1 \\ b_1 \\ \vdots \\ \vdots \\ b_p \end{bmatrix}$$

where  $\{a_i; i = 1, 2, \dots, p\}$  and  $\{b_i; i = 1, 2, \dots, p\}$  are respectively the AR and MA coefficients of the ARMA model.

The innovations representation (see [9] and [10] for descriptive accounts) is equivalent to the Markovian representation in the sense that output means and correlation sequences are the same. The state space equations for the innovations representation are

$$y_t = \hat{y}_t + v_t$$

$$\hat{y}_t = c' x_t$$

$$x_{t+1} = Ax_t + k_t v_t$$

Here  $k_t$  is a Kalman gain vector,  $\{v_t\}_{0}^{T-1}$  is a sequence of iid normal random variables with mean zero and variance  $v_t$ , and  $\hat{y}_t$  is the one-step Kalman prediction based on  $y_0, y_1, \dots, y_{t-1}$ . The Kalman gain satisfies the equations

$$k_t v_t = A[Q_0 - P_t]c + \sigma_w^2 h_1 h_1'$$

where

$$v_t = r_0 - c' P_t c$$

$$P_{t+1} = A P_t A' + k_t k_t' v_t : P_0 = 0,$$

$$Q_0 = A Q_0 A' + \sigma_w^2 h_1 h_1'$$

The matrix  $Q_0$  is the steady-state value of the variance matrix  $P_t$  for the state vector  $x_t$ , and  $r_0$  is the zero-lag value of the correlation sequence for the process  $\{y_t\}$ :

$$\sigma_0^2 = c' A Q_0 A' c + \sigma_w^2 c' h_1 h_1' c$$

The only difference between the innovations model for the signal sequence  $\{y_t\}$  and the innovations model for the noisy sequence  $\{z_t\}$ , with  $z_t = y_t + n_t$ , is in the specification of initial conditions on  $r_0$ :

$$r_0 \longrightarrow r_0 + \sigma_n^2 \delta_0$$

With this representation we may write the joint density function of the observations  $\{z_k\}_0^t$  as

$$f\left\{\left.z_k\right\}_0^t = \prod_{k=0}^{\max(d-1,0)} (2\pi\sigma_n^2)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma_n^2} z_k^2\right\} .$$

$$\prod_{k=\min(d,t)}^t (2\pi v_{k-d})^{-\frac{1}{2}} \exp\left\{-\frac{1}{2v_{k-d}} (z_k - u_k(d) \hat{y}_k(d))^2\right\}$$

Here  $\hat{y}_k(d)$  is the Kalman prediction of  $y_k$ , given that (really, assumed that)  
the ARMA sequence has arrived time  $d \leq k$ :

$$\hat{y}_k(d) = c' x_k(d)$$

$$x_{k+1}(d) = Ax_k(d) + k_{k-d} v_k(d)$$

$$v_k(d) = z_k - \hat{y}_k(d)$$

$$x_k(d) = 0, k < d$$

Of course  $k_t$  follows the recursion previously given.

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This brings up a curious question: does it make sense to speak of time delay estimation in stationary models? In such models the notion of a time origin is irrelevant. It is, however, possible to imagine a stationary field which is fully developed (i.e. stationary), but is only switched into the environment at time  $d$ . This is the case we are considering.

## EXACT AND APPROXIMATE LIKELIHOOD AND DYNAMIC PROGRAMMING

We now introduce a two state machine representation of the TDE problem. We then derive the log likelihood functions which must be maximized to obtain the estimate of the indicator sequence. Exact computation of the likelihood function is shown to be computationally unattractive because it involves computation of the likelihood for each  $d$  in the set  $I_{t+1}$ . As an alternative we compute approximate likelihood on a subset of  $I_{t+1}$  which we denote  $\hat{I}_{t+1}$ .

### Log Likelihood

In the deterministic case the log-likelihood function for  $d$ , based on observations  $\{z_k\}_0^t$  is

$$l_t(d) = \log (2\pi\sigma_n^2)^{-(t+1)/2} - \frac{1}{2\sigma_n^2} \sum_{k=0}^t (z_k - u_k(d)y_{k-d})^2$$

In the stochastic case the log-likelihood is

$$l_t(d) = \sum_{k=1}^t \log \left( \frac{(2\pi(\sigma_n^2 + u_k(d)(v_k(d) - \sigma_n^2)))^{-\frac{1}{2}}}{2(\sigma_n^2 + u_k(d)(v_k(d) - \sigma_n^2))} \right)$$

$$- \sum_{k=1}^t \frac{(z_k - u_k(d)\hat{y}_k(d))^2}{2(\sigma_n^2 + u_k(d)(v_k(d) - \sigma_n^2))}$$

Note that in both cases we may define the log-likelihood in a recursive fashion:

$$l_t(d) = l_{t-1}(d) + p_t(d)$$

The term  $p_t(d)$  is called the path metric.

Because of the nature of the indicator sequence it is clear that for both the deterministic and stochastic signal models the log-likelihood must be evaluated for two separate regions:  $k < d$  and  $k \geq d$ .

This observation suggests a two state machine representation of the TDE problem.

#### The Two State Structure of the TDE Prob

Let 0 and 1 denote the states of the two state machine. See Figure 1. Associate the indicator sequence with a sequence of visits to these states. For  $k < d$  state 0 is visited. For  $k \geq d$  state 1 is visited. An example for  $t = 8$  and  $d = 4$  is illustrated on a two state trellis in Figure 2. Note that transitions from state 1 to state 0 are forbidden because once the indicator is on it stays on (i.e., the signal persists). Thus the TDE problem is that of estimating the time when the transition from state 0 to state 1 occurs.

#### Exact Likelihood

At each time interval associate the log-likelihood  $\ell_t(d>t)$  with state 0.  $\ell_t(d>t)$  is the likelihood that  $d$  is greater than  $t$ . Likewise, associate with state 1 the log-likelihood function  $\ell_t(d\leq t)$ .

The computation of  $\ell_t(d>t)$  is easy. It is dependent only on its most recent value and the path metric  $p_t(d>t)$  where

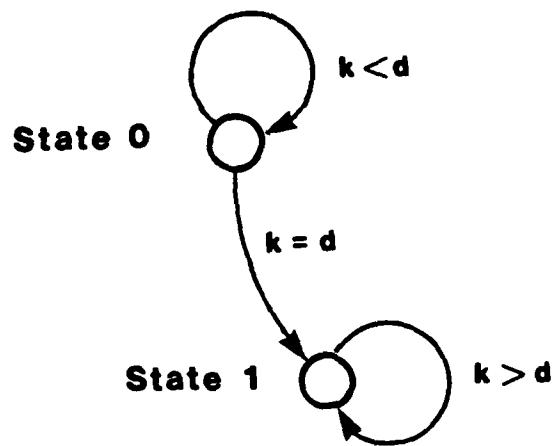
$$p_t(d>t) = \log\left(\frac{1}{2\pi\sigma_n^2}\right)^{\frac{1}{2}} - \frac{1}{2\sigma_n^2} (z_t)^2 .$$

The log-likelihood corresponding to state 1 is more complex.  $\ell_t(d\leq t)$  must be evaluated for every  $d$  in  $I_t$ . This means that computation of the exact likelihood  $\ell_t(d)$  requires a bank of  $t + 2$  filters:  $(t + 1)$  filters to compute  $\ell_t(d\leq t)$  for  $d = 0, 1, \dots, t$ , and one filter to compute  $\ell_t(d>t)$ .

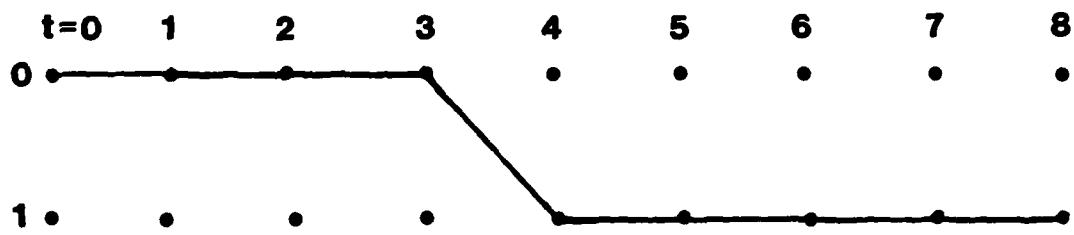
We compute  $\ell_t(d)$  for  $d < t$  as

$$\ell_t(d) = \ell_{t-1}(d) + p_t(d)$$

where



**FIGURE 1 . TWO STATE MACHINE.**



**FIGURE 2 . TWO STATE TRELLIS.**

$d = 4, t = 8$

$$p_t(d) = \log\left(\frac{1}{2\pi\sigma_n^2}\right)^{\frac{1}{2}} - \frac{1}{2\sigma_n^2} (z_t - y_{t-d})^2$$

in the deterministic case and

$$p_t(d) = \log\left(2\pi v_t(d)\right)^{-\frac{1}{2}} - \frac{(z_t - \hat{y}_t(d))^2}{2v_t(d)}$$

for the stochastic signal model. Here  $\hat{y}_t(d)$  is obtained from a one step Kalman predictor initialized at time d. For the case  $d = t$ , we compute  $\ell_t(d)$  as

$$\ell_t(d=t) = \ell_{t-1}(d>t-1) + p_t(d=t)$$

where the path metric depends on which signal model is being used. Having computed the likelihood,  $\ell_t(d)$ , the delay estimate is chosen as

$$\hat{d}_t = \arg\left[\max\left[\ell_t(d>t), \ell_t(d=0), \ell_t(d=1), \dots, \ell_t(d=t)\right]\right]$$

It is clear that, for long data records, computing the likelihood  $\ell_t(d)$  becomes burdensome and impractical. We offer an alternative approach, first introduced in [1].

#### Approximate Maximum Likelihood

In order to avoid the computational problems of simultaneously running a growing bank of filters we compute the likelihood associated with state 1 on a subset of  $I_{t+1}$  which we denote  $\hat{I}_{t+1}$ . The result is that the number of filters required at any time interval is reduced to 3.

Let us examine how the subset,  $\hat{I}_{t+1}$ , is defined. Fix t at 2. At time 2 for state 1 we must consider two possibilities: the signal is just arriving which implies

$$\ell_2(d=2) = \ell_1(d>1) + p_2(d=2);$$

or the signal arrived at time 1 which implies

$$\ell_2(d<2) = \ell_1(d\leq 1) + p_2(d=1)$$

Thus we choose

$$\ell_2(d \leq 2) = \max[\ell_1(d > 1) + p_2(d=2), \ell_1(d \leq 1) + p_2(d=1)].$$

If  $\ell_1(d > 1) + p_2(d=2)$  is chosen, then at  $t = 3$ , we ignore the possibility that the signal arrived at time 1, and do not compute the likelihood for  $d=1$ .

If  $\ell_1(d \leq 1) + p_2(d=1)$  is chosen at  $t = 2$ , then at  $t = 3$  we ignore likelihood for  $d = 2$ . In general the likelihood associated with state 1 is computed as

$$\ell_t(d \leq t) = \max[\ell_{t-1}(d > t-1) + p_t(d=t), \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1})]$$

where  $d_t$  is a running time variable defined as

$$d_t = \begin{cases} t & \ell_{t-1}(d > t-1) + p_t(d=t) > \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1}) \\ d_{t-1}, & \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1}) \geq \ell_{t-1}(d > t-1) + p_t(d=t) \end{cases}$$

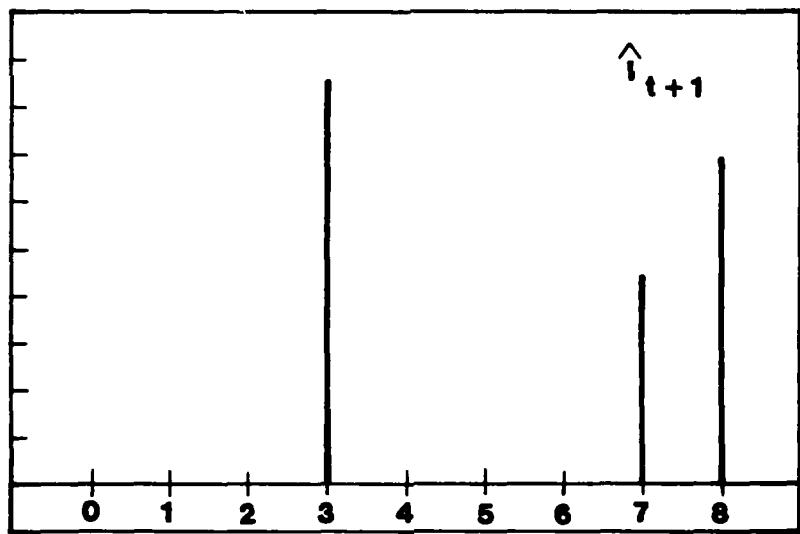
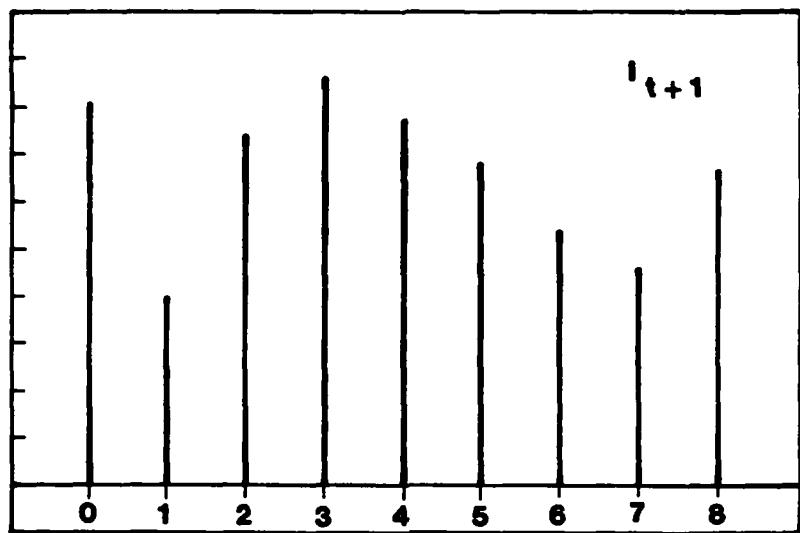
Thus at any time interval we calculate the likelihood of only three of  $t+2$  possible delays in the set  $I_{t+1}$ . The variable  $d_t$  recursively defines  $\hat{I}_{t+1}$ , the subset of  $I_{t+1}$  on which we compute likelihood. At any time  $t$  we may terminate the procedure and compute the final delay estimate by

$$\hat{d}_t = \begin{cases} t+1, & \ell_t(d > t) > \ell_t(d \leq t) \\ d_t, & \ell_t(d \leq t) \geq \ell_t(d > t) \end{cases}$$

Figure 3 illustrates log-likelihoods computed on the set  $I_{t+1}$  and likelihoods computed on the set  $\hat{I}_{t+1}$ , for  $t = 7$ . In the example illustrated, no error would have been made by discarding likelihoods  $\ell_7(0), \ell_7(1), \ell_7(2), \ell_7(4), \ell_7(5),$  and  $\ell_7(6)$ , and saving only  $\ell_7(4), \ell_7(7)$ , and  $\ell_7(8)$ . Such is not always the case.

#### A Note on the Implementation of the TDE Algorithm

The general algorithm is summarized in Table 1. It requires the implementation of three filters at each time interval. Filter 1 computes



**FIGURE 3 . LOG LIKELIHOOD COMPUTED ON  $I_{t+1}$  AND  $\hat{I}_{t+1}$  .**

---

Compute Path Metrics

$$p_t(d=t) \quad p_t(d=d_{t-1}) \quad p_t(d>t)$$

---

Compute Log-Likelihoods

$$\ell_t(d>t) = \ell_{t-1}(d>t-1) + p_t(d>t)$$

$$\begin{aligned} \ell_t(d \leq t) &= \max [\ell_{t-1}(d \leq t-1) + p_t(d=t), \\ &\quad \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1})] \end{aligned}$$

---

Decision

$$d_t = \begin{cases} t, & \ell_{t-1}(d>t-1) + p_t(d=t) > \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1}) \\ d_{t-1}, & \ell_{t-1}(d>t-1) + p_t(d=t) \leq \ell_{t-1}(d \leq t-1) + p_t(d=d_{t-1}) \end{cases}$$

$$\hat{d}_t = \begin{cases} t+1, & \ell_t(d>t) > \ell_t(d \leq t) \\ d_t, & \ell_t(d>t) \leq \ell_t(d \leq t) \end{cases}$$

---

Storage                    Initial Conditions

$$\begin{array}{ll} d_t & \ell_0(d>0) = 0 \\ \ell_t(d>t) & \ell_0(d \leq 0) = 0 \\ \ell_t(d \leq t) & d_0 = 0 \end{array}$$

---

Table 1: Summary of TDE Algorithm

$\lambda_t(d > t)$ , Filter 2 computes  $\lambda_{t-1}(d > t-1) + p_t(d = t)$ , and Filter 3 computes  $\lambda_{t-1}(d \leq t-1) + p_t(d = d_{t-1})$ .

For the deterministic signal models these filters are straightforward and require no comment.

For the stochastic signal however filter 2 and filter 3 require as part of their structure the Kalman predictor.

Let us examine the use of the Kalman predictor within the structure of the TDE algorithm. The Kalman predictor associated with filter 2 is reset to initial conditions at each time interval in order to compute the path metric  $p_t(d = t)$ . The predictor in filter 3 was initialized at time  $d_{t-1}$  and has been running since to give the path metric  $p_t(d = d_{t-1})$ . If the decision  $d_t = t$  is made then we have to reinitialize filter 3 to compute the next path metric  $p_{t+1}(d = d_t = t)$ . Rather than doing this, we can take advantage of the fact that filter 2 has already computed the gain  $k_0$ , and the error variance  $v_0$ . By transferring these variables from filter 2 to filter 3 we save the time required to compute the Kalman gain vector.

## EXPERIMENTAL RESULTS

The thesis of GSF [12] contains simulated examples that show how the trellis evolves and how  $\hat{I}_{t+1}$ ,  $\hat{d}_t$ , and  $\ell_t(d)$  are computed for individual data realizations. The examples are instructive. Here we present only results of Monte Carlo tests

For the deterministic signal model we have defined the SNR as

$$\text{SNR} = 10 \log_{10} \left( \sum_{t=0}^N y_t^2 / \sigma_n^2 \right)$$

where  $N = T-d$  is the number of samples of the signal in the observation interval. The signal model is simply  $y_t = u_t$  (the unit step sequence).

To demonstrate algorithm performance when the underlying signal may be modeled as an ARMA structure driven by white noise, an ARMA (3,2) model was chosen. The z-transfer function of the system is

$$H(z) = \frac{1 - 1.750z^{-1} + .800z^{-2}}{1 - 1.500z^{-1} + 1.210z^{-2} - .455z^{-3}}$$

For the stochastic signal we have defined the SNR as

$$\text{SNR} = 10 \log_{10} \left[ \frac{N r_o}{\sigma_n^2} \right],$$

where  $N$  is the number of samples of the signal in the observation interval:

$$N = T-d.$$

The Monte Carlo tests consisted of running the algorithm 1000 times. Each time the delay was fixed at 14 and the length of the simulation was 30 ( $t = 0, 1, \dots, 29$ ). The final delay estimate from each simulation is stored. These results are then averaged to compute the sample statistics of the delay estimator.

A typical relative frequency of delay estimates is illustrated in Figure 4. The signal is a unit pulse sequence and the output SNR is 15 db.

Table 2 summarizes the results of the Monte Carlo tests for the deterministic signal model. Figure 5 is a plot of the MSE vs. SNR obtained from the Monte Carlo tests using the deterministic unit pulse sequence. Figure 6 shows relative frequency of delay estimates for the stochastic case when output SNR = 21.7. Table 3 summarizes the results of the Monte Carlo tests using the stochastic signal model. Figure 7 is a plot of the MSE vs. SNR for the stochastic signal model. Figure 8 is a plot of the miss probability vs. SNR for both the deterministic and stochastic model. The miss probability is defined to be the probability that the delay is estimated to be greater than or equal to  $T-1$ , when in fact the signal was present at delay  $d \leq T-1$  (in this case  $d=14$ ).

The use of "soft" decision would offer an attractive method for improving the algorithm performance. At present each time the decision  $d_t = t$  is made we reinitialize the filter associated with state 1 and eliminate past values of  $d$  from consideration. Instead, if the decision  $d_t = k$  is made and this value persists for  $n$  steps, then a filter should be maintained for computing  $\ell_t(d=k)$ . This increases the size of the subset of  $I_{t+1}$  on which we compute  $\ell_t(d)$ . The value of  $n$  is chosen to get a reasonable trade off between computation time and estimation accuracy.

MONTE CARLO SIMULATION FOR THE RELATIVE  
 FREQUENCY OF DELAY ESTIMATES

ACTUAL DELAY- 15.00 SNR- 15.00  
 SAMPLE VARIANCE- 5.34 SAMPLE MEAN- 14.07  
 NUMBER OF RUNS- 1000.00 MSE- 6.28

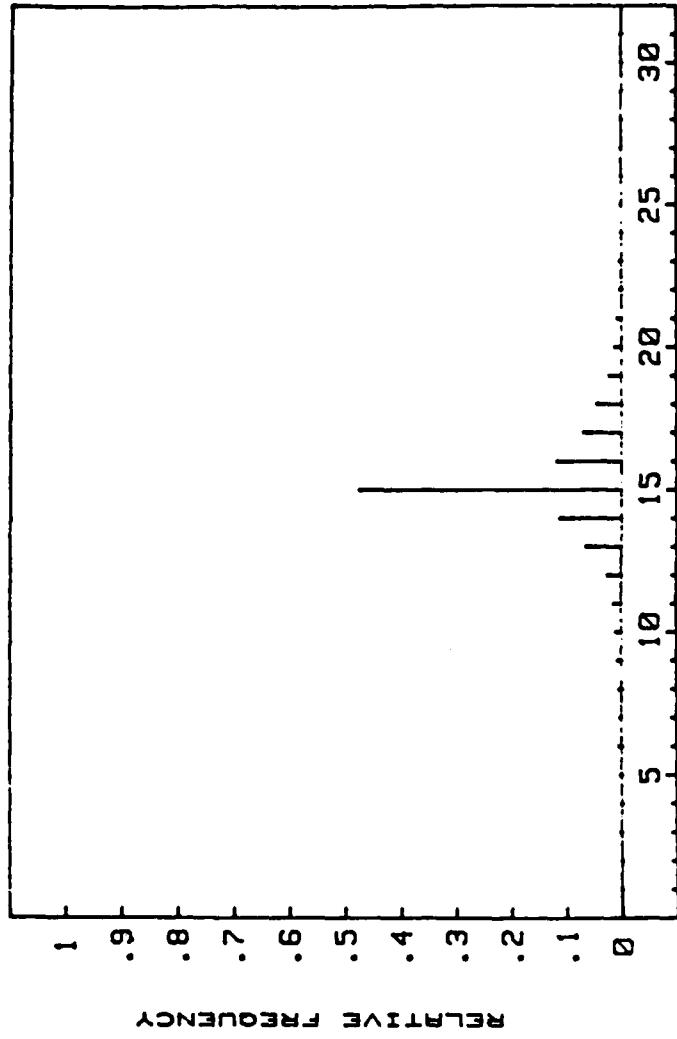


Figure 4. Relative frequency of delay estimates.  
 Signal: Unit pulse.

Unit Pulse		
SNR	MSE	Bias
20	1.65	.02
17	2.44	.04
15	4.82	.07
12	18.12	.29
10	22.74	.38
6	46.15	.89

Table 2. Summary of Monte Carlo Tests for the Unit Pulse and Modulated Pulse Signals

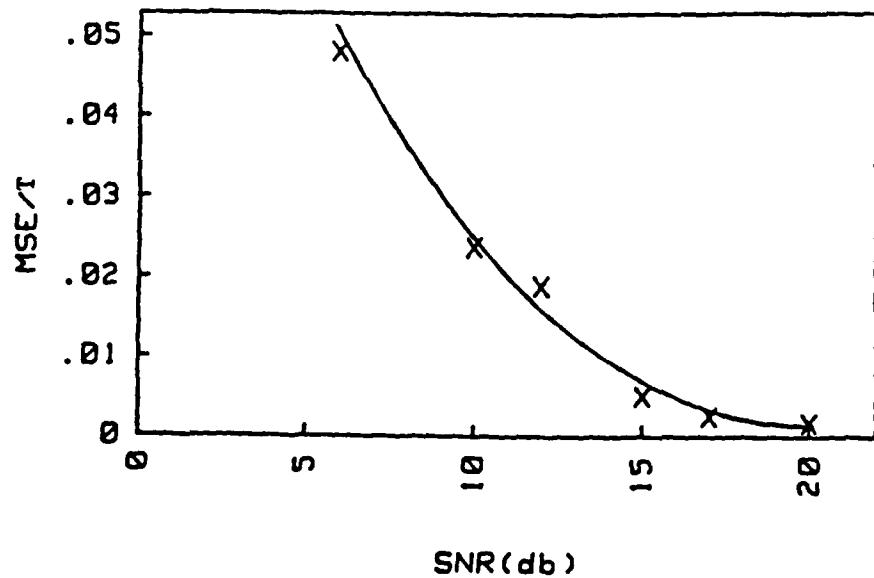


Figure 5. MSE/T vs. SNR for Unit Pulse Signal.

MONTE CARLO SIMULATION FOR THE RELATIVE  
FREQUENCY OF DELAY ESTIMATES.

ACTUAL DELAY = 15.00      SNR = 21.70  
SAMPLE VARIANCE = 8.87      SAMPLE MEAN = 16.14  
NUMBER OF RUNS = 1000.00      MSE = 9.36

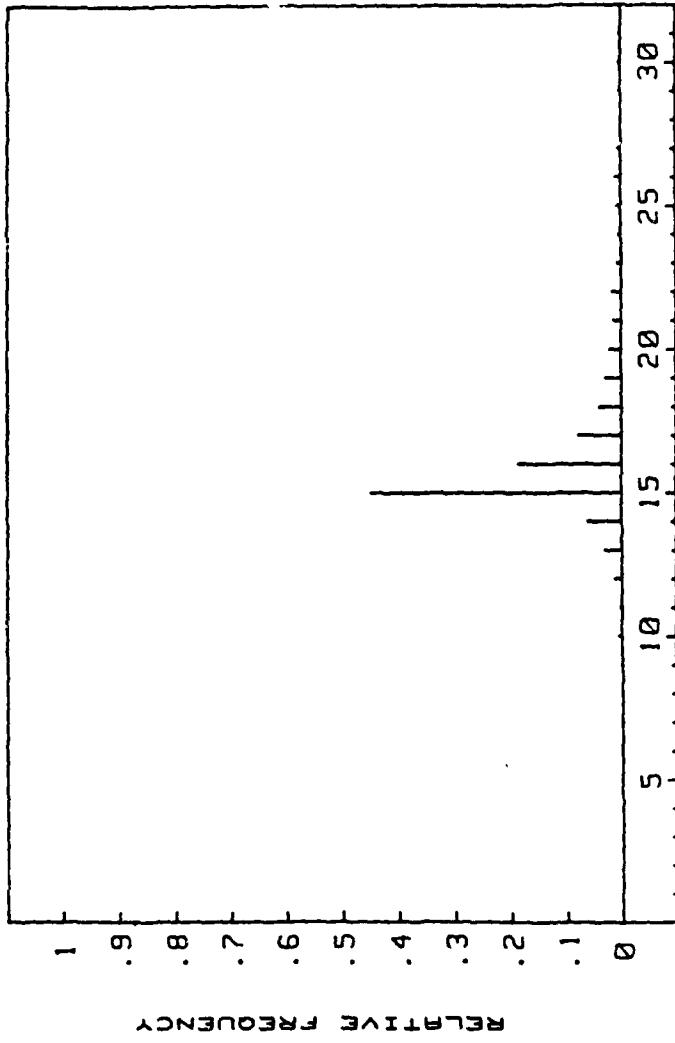


Figure 6.  
DELAY ESTIMATES  
Relative frequency of delay estimates.  
Signal: Stochastic ARMA.

Table 3      Summary of Monte  
Carlo Tests for the  
Stochastic Signal

SNR	MSE	Bias
31.70	.49	.20
28.70	.84	.28
26.70	1.47	.37
23.70	5.20	.85
21.70	9.36	1.14
17.70	39.05	3.31

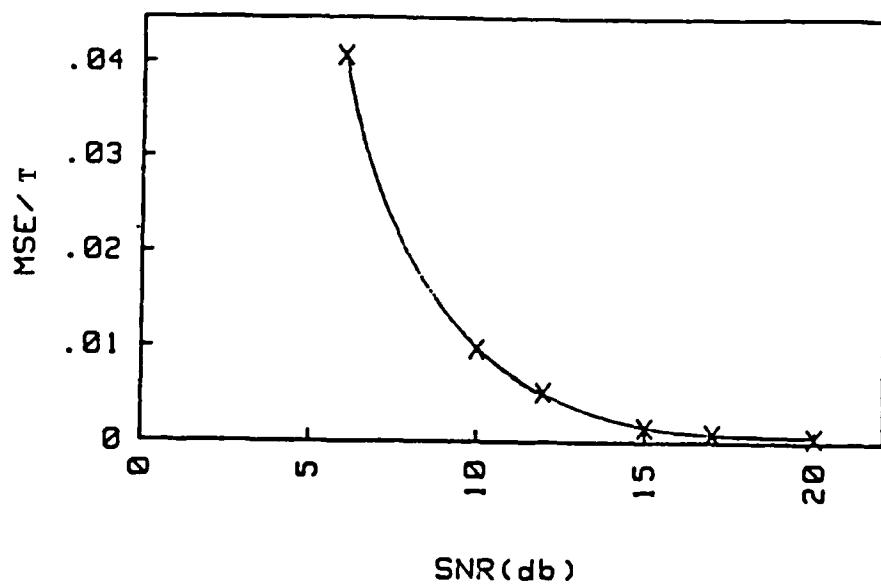
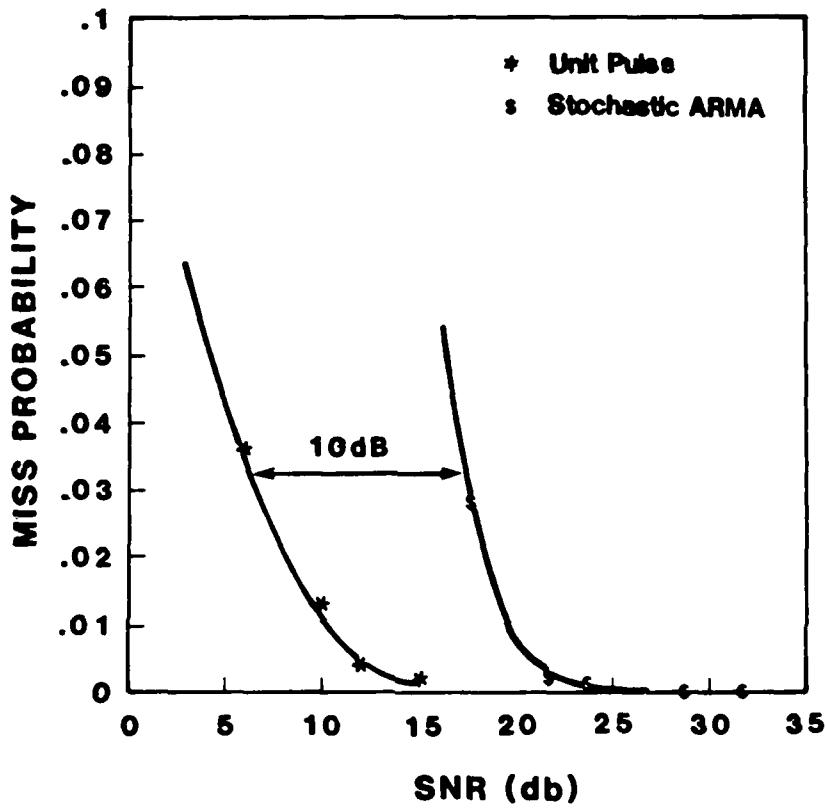


Figure 7. MSE/T vs. SNR for Stochastic ARMA Signal.



**FIGURE 8 . MISS PROBABILITY VS. SNR.**

## REFERENCES

1. IEEE Transactions, "Acoustics, Speech and Signal Processing," Special Issue on Time Delay Estimation, Vol. 29, June, 1981.
2. A.S. Willsky et al., "Dynamic Model Based Techniques for the Detection of Incidents on Freeways," IEEE Trans on Auto. Control, Vol. AC-25, June 1980.
3. P.M. Newbold, Yu-Chi Ho, "Detection of Changes in the Characteristics of a Gauss-Markov Process," IEEE Trans. on Aerospace and Electronics, Vol. 4, No. 5, Sept. 1968.
4. R.S. Iyer, "Arrival Times of Satellite Broadened Laser Pulses," IEEE Trans. Aerospace and Electronic Systems, Vol. 12, No. 5, Sept. 1976
5. R.K. Mehra and J. Peschon, "An Innovations Approach to Fault Detection and Diagnosis in Dynamic Systems," Automatica, Vol. 7, pp. 637-640, 1971.
6. A.S. Willsky and H.L. Jones, "A Generalized Likelihood Ratio Approach to the Detection and Estimation of Jumps in Linear Systems," IEEE Trans. Auto. Control, p. 108, February 1976.
7. R.J. Kenefic, "A Bayesian Approach to Time Delay," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 29, No. 3, June 1981.
8. R. Singer et al., "Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multi-Target Environments," IEEE Trans. Info. Theory, Vol. IT-20, pp. 423-432, July 1974.
9. B.D.O. Anderson, J.B. Moore, "Optimal Filtering," Prentice-Hall, 1979.
10. C. Gueguen, L.L. Scharf, "Exact Maximum Likelihood Identification of ARMA Models: A Signal Processing Perspective," Proc. EUSIPCO, Lusanne, Switzerland, Sept. 1, 1980.
11. L.L. Scharf, G.K. Lee, G.J. Fetzer, "TDE Using Dynamic Programming," Proc. MIMI, May 1980.
12. G.J. Fetzer, "Dynamic Programming for Time Delay Estimation of Deterministic and Stochastic Signals," M.S. Thesis, Colorado State University, Ft. Collins, CO (Summer 1982).

## REFERENCES

1. IEEE Transactions, "Acoustics, Speech and Signal Processing," Special Issue on Time Delay Estimation, Vol. 29, June, 1981.
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## REFERENCES

1. IEEE Transactions, "Acoustics, Speech and Signal Processing," Special Issue on Time Delay Estimation, Vol. 29, June, 1981.
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3. P.M. Newbold, Yu-Chi Ho, "Detection of Changes in the Characteristics of a Gauss-Markov Process," IEEE Trans. on Aerospace and Electronics, Vol. 4, No. 5, Sept. 1968.
4. R.S. Iyer, "Arrival Times of Satellite Broadened Laser Pulses," IEEE Trans. Aerospace and Electronic Systems, Vol. 12, No. 5, Sept. 1976
5. R.K. Mehra and J. Peschon, "An Innovations Approach to Fault Detection and Diagnosis in Dynamic Systems," Automatica, Vol. 7, pp. 637-640, 1971.
6. A.S. Willsky and H.L. Jones, "A Generalized Likelihood Ratio Approach to the Detection and Estimation of Jumps in Linear Systems," IEEE Trans. Auto. Control, p. 108, February 1976.
7. R.J. Kenefic, "A Bayesian Approach to Time Delay," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. 29, No. 3, June 1981.
8. R. Singer et al., "Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multi-Target Environments," IEEE Trans. Info. Theory, Vol. IT-20, pp. 423-432, July 1974.
9. B.D.O. Anderson, J.B. Moore, "Optimal Filtering," Prentice-Hall, 1979.
10. C. Gueguen, L.L. Scharf, "Exact Maximum Likelihood Identification of ARMA Models: A Signal Processing Perspective," Proc. EUSIPCO, Lusanne, Switzerland, Sept. 1, 1980.
11. L.L. Scharf, G.K. Lee, G.J. Fetzer, "TDE Using Dynamic Programming," Proc. MIMI, May 1980.
12. G.J. Fetzer, "Dynamic Programming for Time Delay Estimation of Deterministic and Stochastic Signals," M.S. Thesis, Colorado State University, Ft. Collins, CO (Summer 1982).

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